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**ROTATIONAL SEISMOMETER**

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### ABSTRACT

Elastic wave theory predicts that the elastic waves generated by an earthquake and other seismic events include a measurable component of horizontal elastic rotation. Several rotational seismometers have been designed to measure this rotational component.

The first design consisted of a large inertial ring of mercury terminated in a coaxial set of stand pipes housing capacitive transducers to sense the displacement of the mercury in the stand pipes.

The second design consisted of a continuous inertial ring of water with a thermistor transducer to detect flow within the pipe.

The third design consisted of a continuous inertial ring of water with a thin paddle placed in the path of flow. Semiconductor strain gages were used to detect displacement of the paddle due to flow.

Some of the designs proved to be sensitive to seismic disturbances but none were particularly sensitive to horizontal rotation,

Problems encountered involved spurious signals due to a "flexible" pipe, electronic noise and drift, temperature stabilization, and moisture penetration into the transducers.

The capacitive transducer seemed to be the most promising, and experimentation is continuing using this transducer within a rigid steel pipe.

### SUMMARY

It is the purpose of this report to present the final status and a comprehensive review of the scientific investigations conducted on rotational seismometers under Contract No. F19628-68-C-0370, sponsored by The Advanced Research Projects Agency of the Department of Defense.

A rotational seismometer is a device designed to measure directly the rotational phases of seismic disturbances. There are two scientific benefits which possibly could result from operating such a device:

1. Improvement in the accuracy of reading the S-wave arrival time. This would allow the S-wave travel time curves to be improved, which would in turn result in a better knowledge of deep earth structure.

2. Direct measurement of an elastic wave parameter which is source dependent. This could be useful in studying earthquake mechanism and in distinguishing between earthquakes and underground nuclear explosions.

Several rotational seismometers of unique design have been constructed and operated in the Byerly Seismographic Station located in Strawberry Canyon on the University of California campus.

## INTRODUCTION

Elastic wave theory predicts that the elastic waves generated by an earthquake or similar event include a component of horizontal elastic rotation,  $\Omega_z$ , where

$$\Omega_z = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad (1)$$

$\Omega_z$  has the geometrical interpretation shown in Fig. 1.

This project is an attempt to design and construct a rotational seismometer which will detect the component of horizontal elastic rotation and will be insensitive to other phases of seismic disturbances.

There are two scientific benefits which possibly could result from operating such a device:

1. Improvement in the accuracy of reading the S-wave arrival time. This would allow the S-wave travel time curves to be improved, which would in turn result in a better knowledge of deep earth structure.
2. Direct measurement of an elastic wave parameter which is source dependent. This could be useful in studying earthquake source mechanism and in distinguishing between earthquakes and underground nuclear tests.

## DESCRIPTION OF THE ROTATIONAL SEISMOMETERS

The first rotational seismometer, constructed in the Byerly Seismographic Station located in Strawberry Canyon on the University of California campus, consisted essentially of a large inertial ring

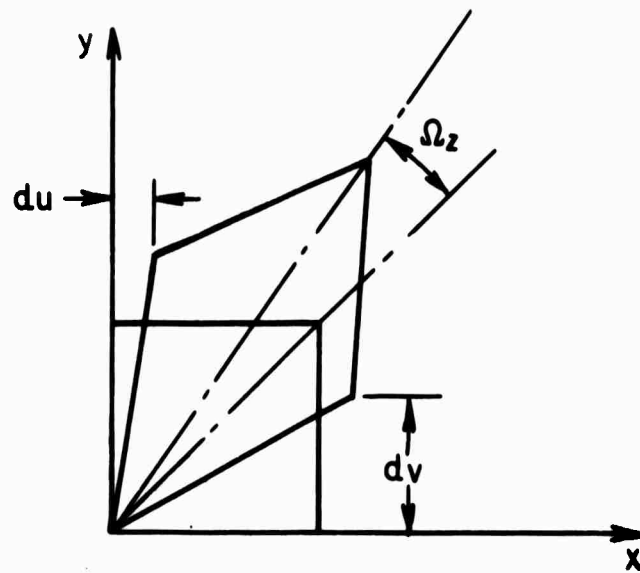


FIG.1 GEOMETRICAL INTERPRETATION OF  
ELASTIC ROTATION

of mercury terminated in a coaxial set of standpipes as shown in Fig. 2.

Angular acceleration,  $\ddot{\Omega}_z$ , produces a height differential between the two columns of mercury in the standpipes. The height differential is sensed electronically and recorded. The salient feature of this type of rotational seismometer is the design of the standpipes and the associated displacement transducer. A cross section is shown in Fig. 3. As can be seen, the two columns of mercury, which are coaxial inside the standpipes, connect with each other through the inertial ring. The purpose of the standpipes is to provide both a restoring force and a means of detecting relative motion of mercury with respect to the body of the ring. This appears as a height differential in the two columns. This height differential is sensed by the change in capacity between the mercury and the pair of coaxial capacitor plates,  $P_1$  and  $P_0$ , directly above the mercury. The reason for using the coaxial standpipes then becomes clear. The purpose is to make the device insensitive to tilt. Regardless of the axis of tilting, the increase in capacity on one side of either capacitor plate is exactly cancelled by the decrease on the other side (assuming small tilt angles). See Fig. 4 for an illustration of this.

Another feature of the transducer is that the areas of the two columns of mercury are exactly equal and the volume above them is completely filled with oil. This guarantees that the null position of the mercury will remain fixed so that a height increase in one column will always be accompanied by an identical decrease in height in the

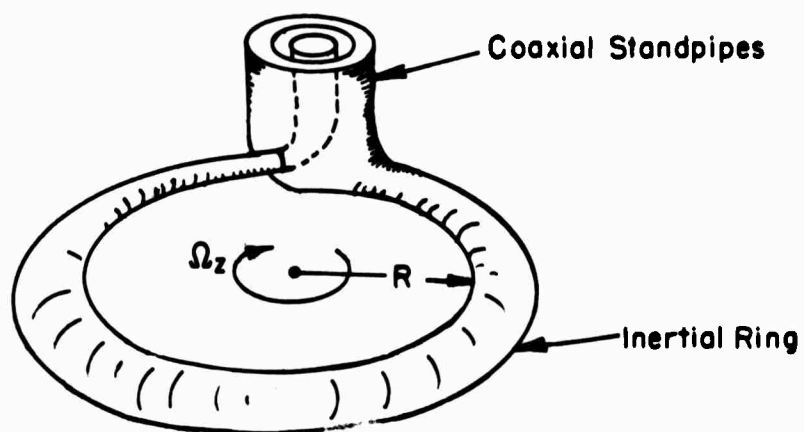


FIG. 2 ROTATIONAL SEISMOMETER

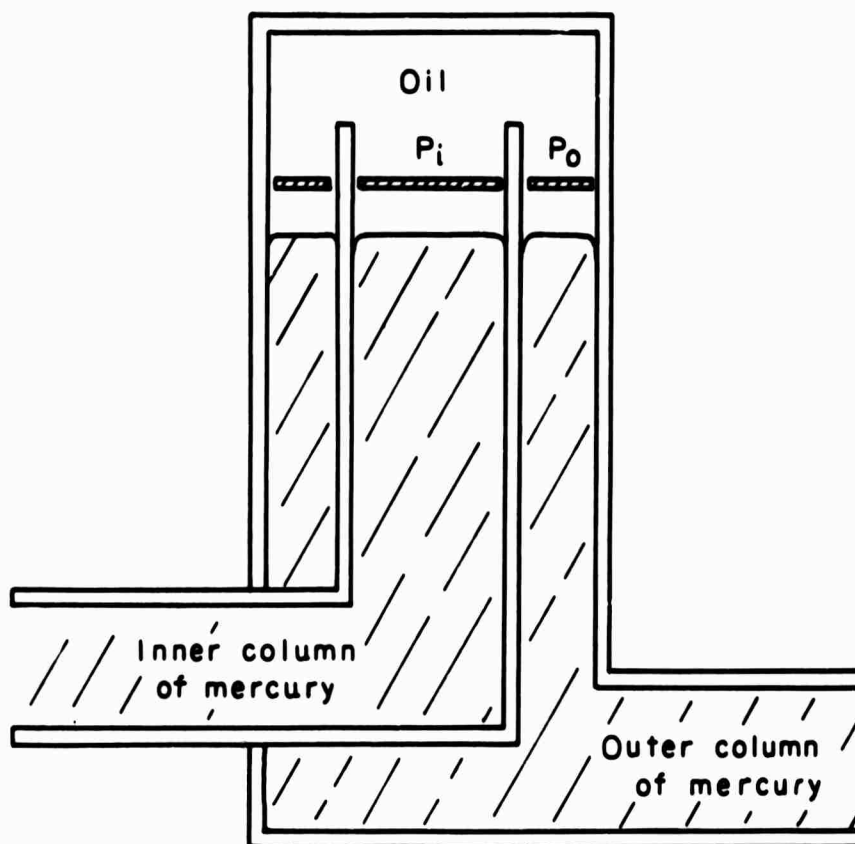


FIG. 3 VIEW OF TRANSDUCER SHOWING STANDPIPES  
AND CAPACITOR PLATES



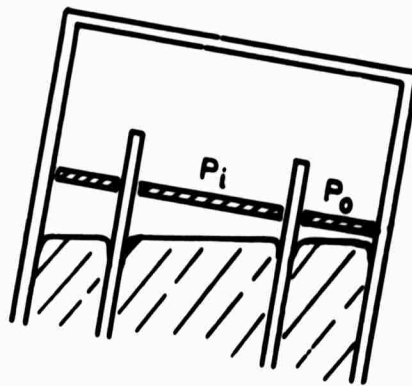


FIG 4 EFFECT OF TILTING

other. It will be seen later that it is important to maintain the stability of the null position of the mercury because the gain of the capacitance bridge used to sense the changes in capacity depends inversely on the quiescent separation between the mercury and the capacitor plates. In fact, the greatest problems encountered involved the instability of this null position.

The relative displacement of the two columns of mercury is sensed by a simple capacity bridge driven by the reference voltage of a synchronous detector. When a differential change in height takes place, the capacity of one plate relative to the mercury decreases while that of the other increases. This unbalances the bridge generating a signal which is fed into the synchronous detector. In this case the synchronous detector is actually part of a Princeton Applied Research JB-5 Lock-In Amplifier. The block diagram of the electronics is shown in Fig. 5.

#### THEORY OF THE ROTATIONAL SEISMOMETER

If rotation,  $\Omega_z$ , of the seismometer due to a seismic disturbance is assumed to be sinusoidal in form:

$$\Omega_z = \Omega_{z_{\max}} \sin \omega t \quad (2)$$

then the displacement sensitivity of the seismometer may be expressed:

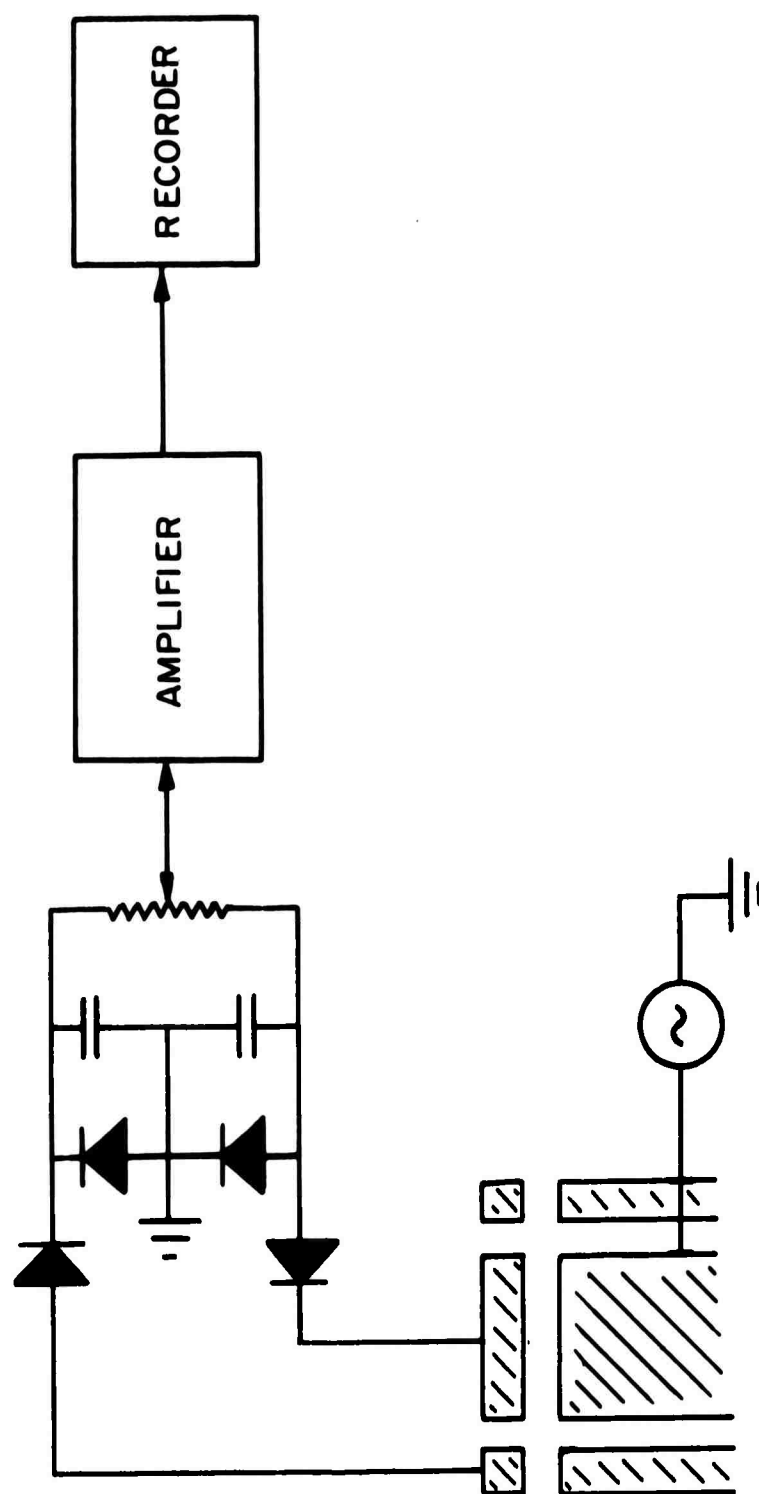


FIG 5 ELECTRONIC BLOCK DIAGRAM

$$\frac{z_{\max}}{\ddot{\Omega}_{z_{\max}}}(\omega) = \frac{\omega^2 R^2}{\frac{g}{2\pi R} - \omega^2 + 4 \frac{g}{2\pi R} \omega^2} \quad (3)$$

The calculations are carried out in Appendix A.

### SENSITIVITY CALCULATIONS

Consider a remote event. Conservatively assume that minimum elastic rotation  $\ddot{\Omega}_z$  to be detected is of the order of  $10^{-10}$  radians and that it has a period of  $2\pi$  seconds. Then from Eq. (2),  $\ddot{\Omega}_{z_{\max}} \approx 10^{-10} \text{ rad/sec}^2$ . Set  $R = 1$  meter. Then from Eq. (3)

$$z_{\max} = \frac{2\pi R^2}{g} \ddot{\Omega}_{z_{\max}} \approx \frac{2\pi (1)}{9.8} \cdot 10^{-10} \approx \text{Å} \quad (4)$$

If the quiescent separation between the plates and the mercury is 1 mm and the voltage applied to the bridge is 200 volts peak, from Eq. (5B), Appendix B

$$E_o = \frac{V_p}{2d_o} z_{\max} = \frac{200}{2 \times 10^{-3} \text{ m}} \times 10^{-10} \text{ m} \approx 10 \mu\text{-volts} \quad (5)$$

This is certainly a detectable signal level especially using a system of coherent detection such as the JB-5.

### EQUIPMENT

A photograph of the actual transducer which was constructed for the rotational seismometer is shown in Fig. 6. The two coaxial stand-pipes each have a cross-section area of  $\pi$  square inches. The top of the transducer is removed and lies on its side showing the perforated coaxial capacitor plates which are mounted on it.

A photograph of the seismometer set up in the seismic vault is shown in Fig. 7. A ten foot square wooden platform supports the seismometer. The flexible plastic tube ( $2R \approx 7$  feet) has been wrapped with nylon filament tape to add to strength and rigidity. The transducer sits in a shallow pan as a safety precaution in case of mercury spills. Behind the seismometer may be seen a signal generator which was operated at 100 kHz, an oscilloscope, an amplifier, and a Geotech Helicorder.

### SEISMOMETER OPERATION

In the early summer of 1968 the rotational seismometer was constructed in the seismic vault of the Byerly Seismographic Station located in Strawberry Canyon on the University of California campus. It was filled with 380 pounds of mercury and first placed in operation on July 9, 1968. The output of the transducer was demodulated, passed through a low-pass filter, and recorded on a Geotech Helicorder.

Several large teleseisms (see Fig. 8) and numerous small local earthquakes were recorded. A Nevada nuclear test was also recorded. Preliminary comparisons of the rotational seismogram with conventional horizontal seismograms of the same event revealed a greater P-wave

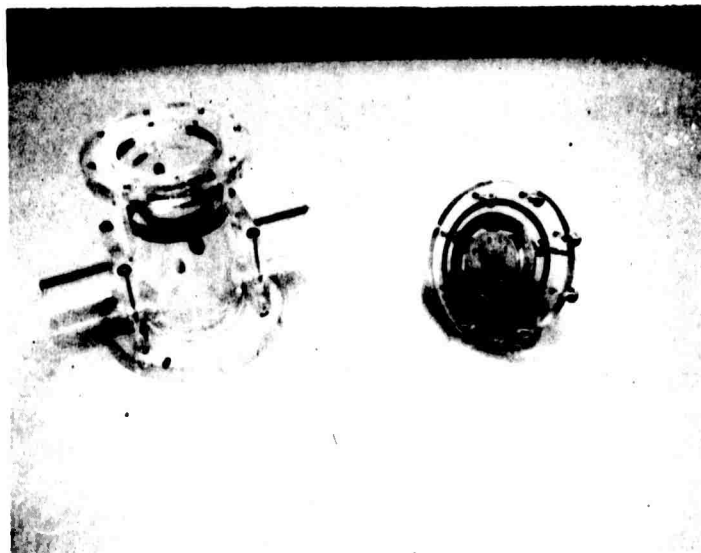


FIG 6 PHOTOGRAPH OF TRANSDUCER

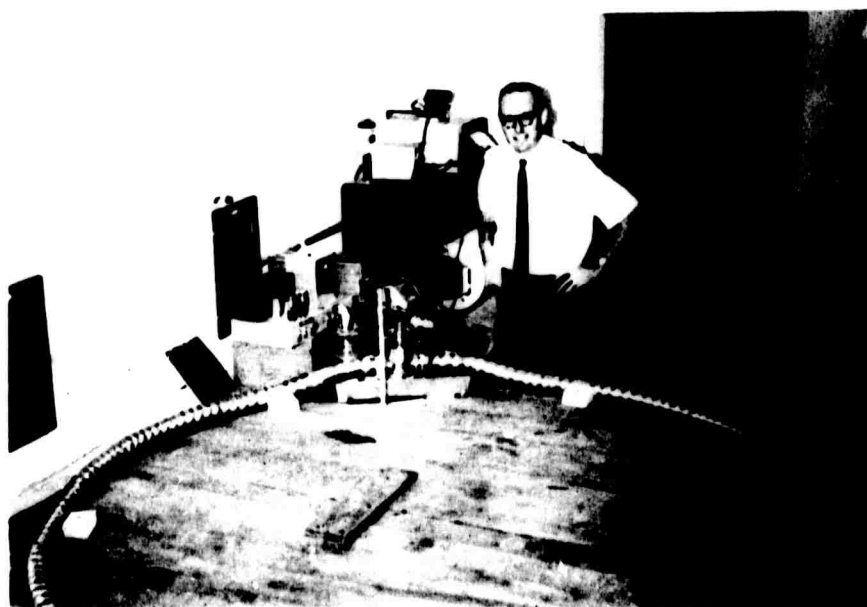


FIG 7 PHOTOGRAPH OF ROTATIONAL SEISMOMETER

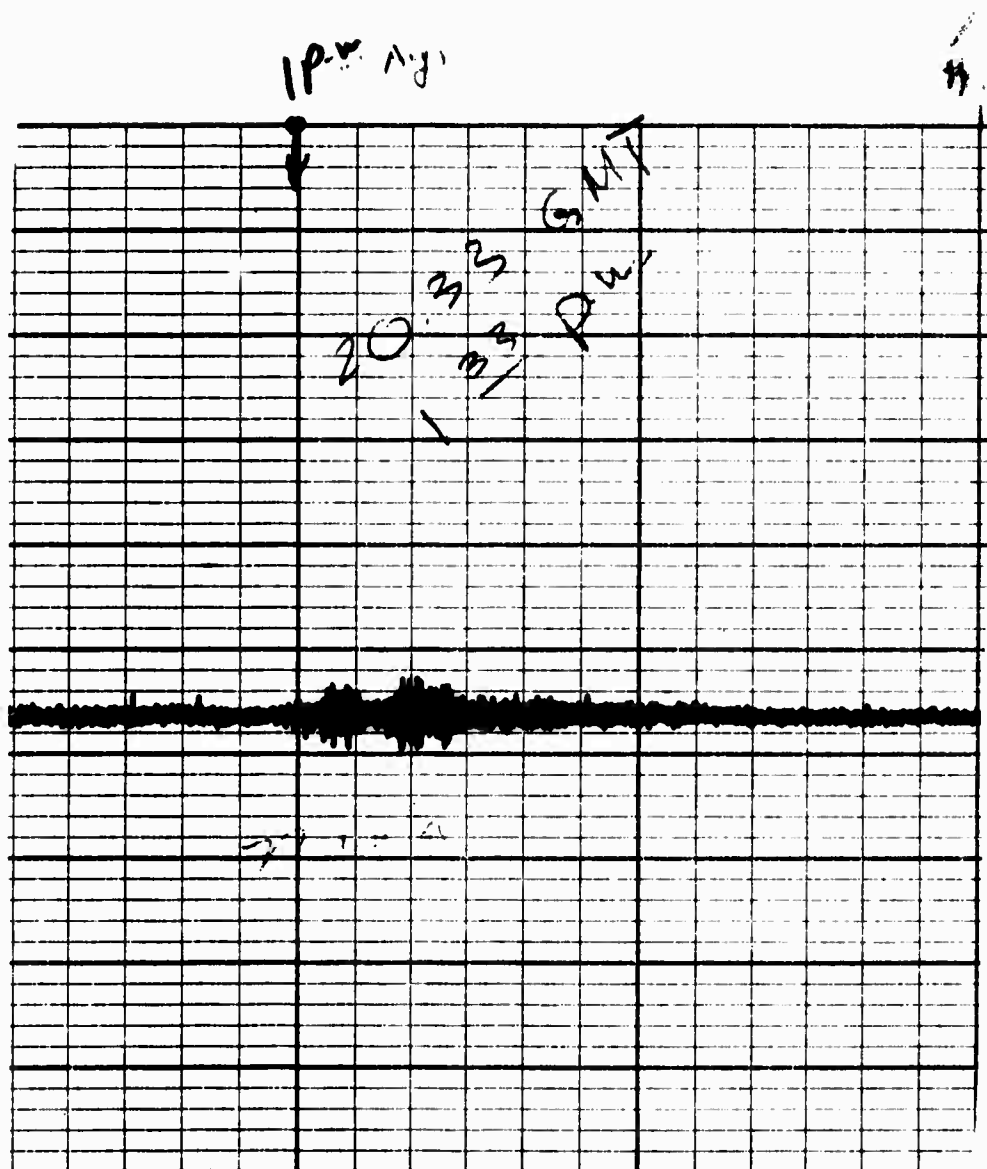


FIG 8 RECORDING OF TELESEISM FROM A SOUTH PACIFIC EARTHQUAKE

sensitivity than predicted. Furthermore, the output of the transducer showed considerable time-dependent drift away from the null position toward unbalance and decreased sensitivity.

After investigation the following problems became evident:

Problem 1. Plastic tube exhibits "creep."

Under the pressure of a few inches of mercury, the plastic tube "creeps" causing the volume of the tube to slowly increase as a function of time. This causes the quiescent distance  $(d_0)$  between the surface of the mercury and the capacitor plates in the transducer to increase. Sensitivity is proportional to the inverse of  $d_0$ . In about two days of operation, sensitivity decreases to such an extent as to make the recording of seismic disturbances impossible.

Problem 2. Flexible tube's reaction to seismic disturbances results in volume changes.

A compressional pulse or wave incident on the round flexible tube perpendicular to its axis will cause the round tube to deform. This deformation causes a change in the internal volume of the tube. Furthermore, this deformation may be propagated along the tube.

The above mentioned problems have the effect of modulating the actual signal output because of the effects of Problem 3.

Problem 3. The two capacitive arms of the bridge in the transducer are not identical.

One capacitor is a ring and the other capacitor is a disc. This results in differences in the fringing effects. The circumference of fringing in the ring capacitor is:



$$C_{\text{ring}} = 2\pi(1.2 + 1.5625) = 17.4 \text{ inches.}$$

The circumference of fringing in the disc capacitor is:

$$C_{\text{disc}} = 2\pi(1) = 6.28 \text{ inches.}$$

Furthermore, the disc electric field is contained within and confined by the ring electric field. Fringing effects should be negligible if  $d_o$  is constant. But once the electronics system is balanced, if  $d_o$  changes, then the electronics system becomes unbalanced. As explained by Problems 1 and 2,  $d_o$  does change.

#### SOLUTIONS TO THE PROBLEMS

To overcome the problems associated with a flexible "creeping" plastic pipe, two additional seismometers were constructed.

The first of these rotational seismometers was a six inch standard steel pipe laid out in a circle approximately 18 inches in diameter, and filled with water. Two 0.02 inch glass probe thermistors (Victory Engineering Corporation; ZB48A23: 75 Kohm) provided with stainless steel extension sleeves and teflon insulated leads, were placed one and five-eighths inches below the top of the pipe, one quarter of an inch apart on a line parallel with the axis of the pipe. A 22 ohm, 1 watt resistor was placed two and one-half inches directly below the thermistors and centered between them. The resistor is heated so that a convection pattern is set up centered between the thermistors. The thermistors were connected to a resistive bridge excited by the Princeton Applied Research HR-8 lock-in amplifier. Figure 9 shows the

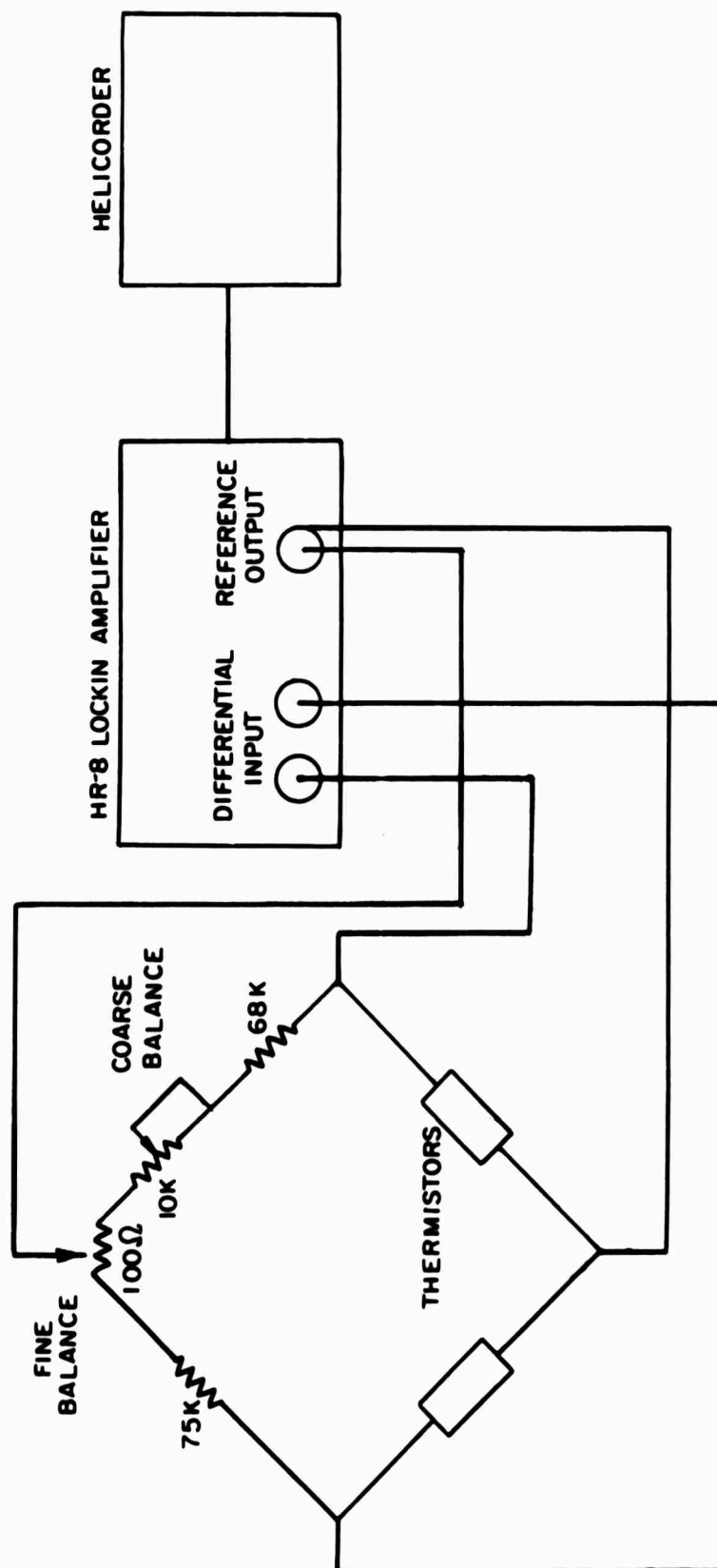


FIG 9 BRIDGE AND BLOCK DIAGRAM OF THE ELECTRONICS  
USED FOR THE THERMISTOR TRANSDUCER

bridge used and a block diagram of the electronics used. If the convection pattern is moved either upstream or downstream one thermistor will experience a rise in temperature and the other will experience a fall in temperature. The proper connections to the bridge will add the output signals of the two thermistors and at the same time provide some temperature compensation.

The second rotational seismometer was a one and one-half inch standard steel pipe laid out in a circle approximately six feet in diameter, and filled with water. A paddle made from high impact polystyrene 0.02 inches thick was inserted in the pipe perpendicular to the direction of flow. The polystyrene was chosen because its density is near that of water, its water absorption coefficient is very low (less than 0.01%), and its modulus of elasticity is very high for plastics (greater than  $4 \times 10^5$  psi). The paddle was one and one-half inches in diameter leaving approximately a one-sixteenth inch clearance with the walls of the pipe. (The standard one and one-half inch pipe is actually 1.61 inches inside diameter.) Two Baldwin-Lima-Hamilton semiconductor strain gages (SPB3-20-100 C9) with nominal resistance of 1000 ohms and gage factor of 140 were attached, one on each side of the neck of the paddle. The gages were incorporated into a standard bridge circuit. Since one gage will be in compression when the other is in tension, subtracting their outputs will double the signal output of the bridge and at the same time provide some temperature compensation. Figure 10 shows the bridge used and a block diagram of the electronics used. Figure 11 shows a photograph of the polystyrene paddles with the semiconductor strain gages attached.

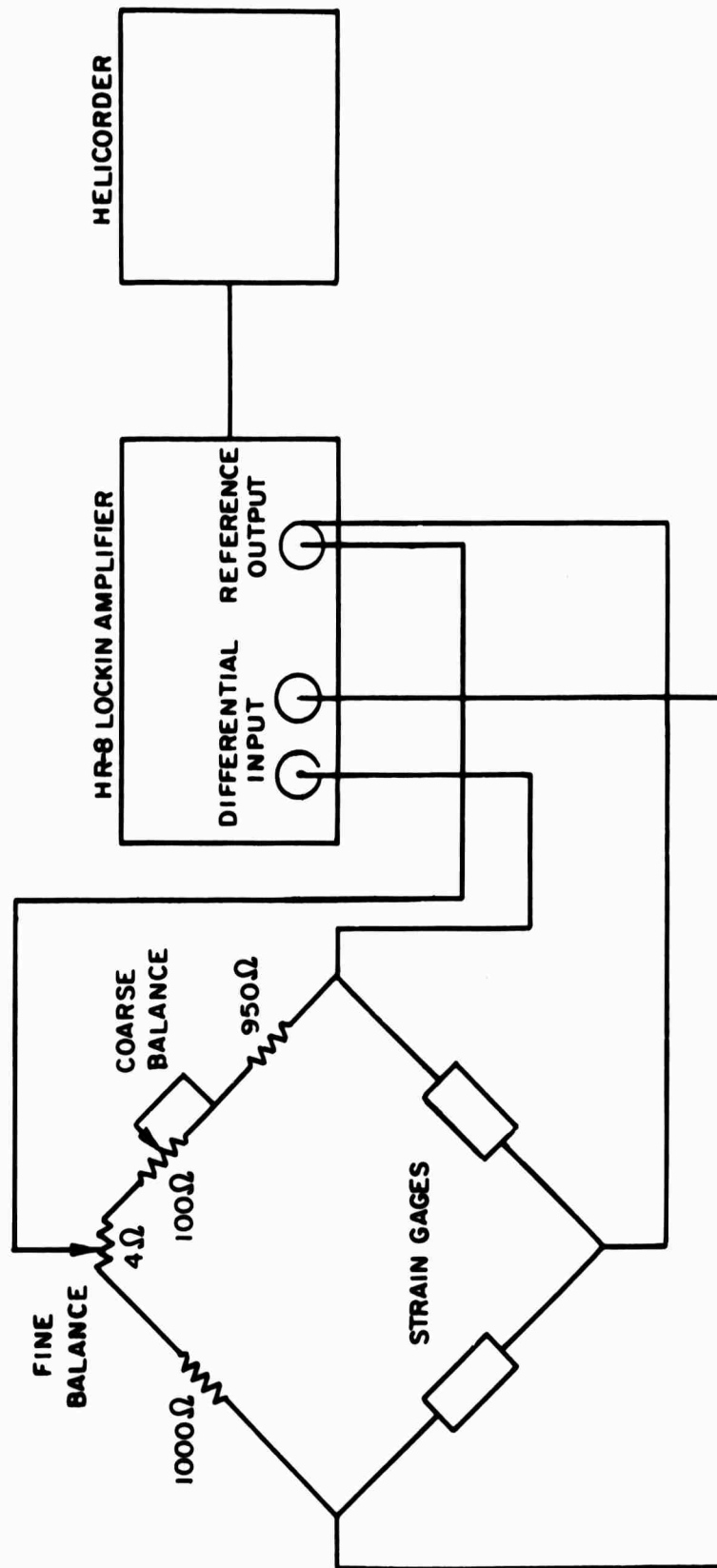


FIG. 10 BRIDGE AND BLOCK DIAGRAM OF THE ELECTRONICS  
USED FOR STRAIN GAGE TRANSDUCER

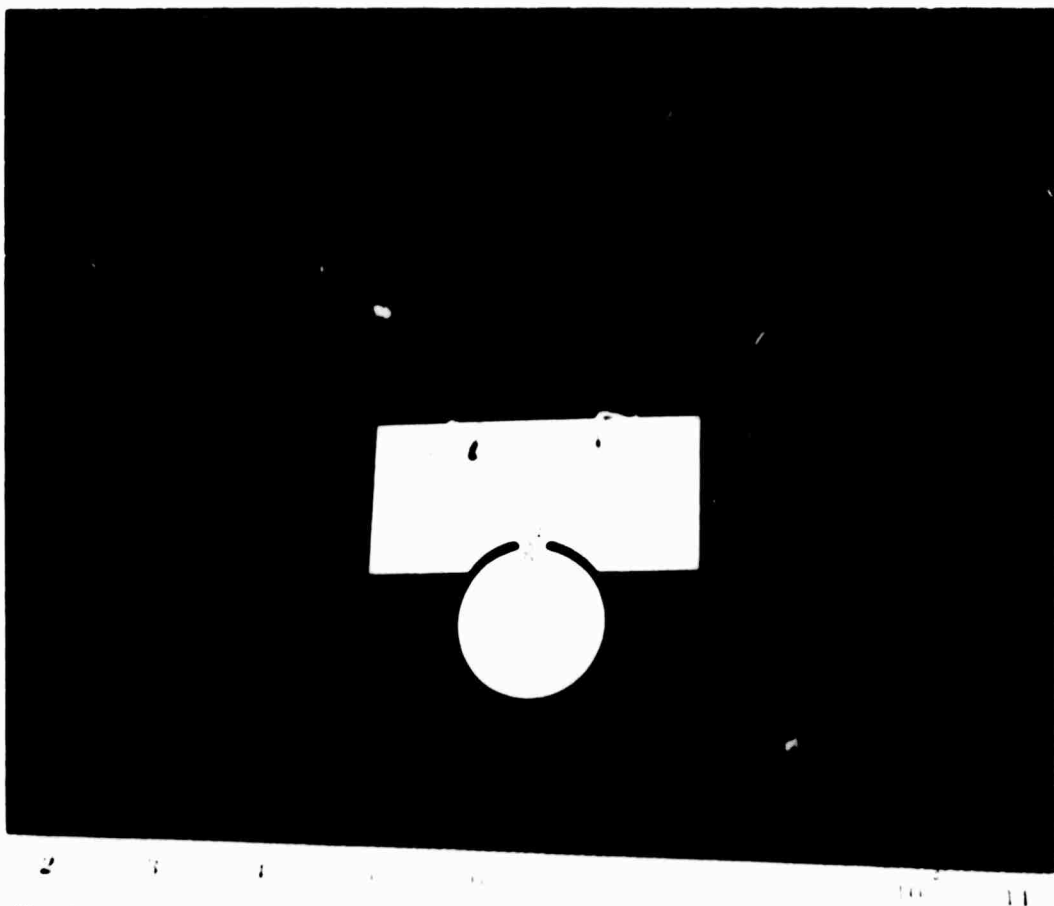


FIG.II POLYSTYRENE PADDLES WITH STRAIN GAGES ATTACHED.  
TWO SMALL PINS MAY BE SEEN FOR CENTERING THE  
PADDLE WITHIN THE PIPE .

A laboratory experiment was conducted to illustrate the sensitivity of a semiconductor strain gage paddle transducer. A paddle made of 1/32 inch lucite with one strain gage attached was placed in a short length of two-inch lucite pipe. The pipe was corked at each end with inlet and outlet of 9 mm glass tubing. Water was supplied at one end from a reservoir and allowed to drip from the other end. The tube was completely filled with water, allowed to stand 24 hours, and all air bubbles were drawn off.

The single strain gage was connected to a simple resistive bridge excited by a Princeton Applied Research HR-8 lock-in amplifier. The transducer proved to be very sensitive to disturbances in established flow patterns. Each time a drop of water broke off from the glass tube at the draining end, the HR-8 amplifier indicated a change in bridge output of several microvolts.

This transducer cannot be meaningfully calibrated because (1) the very slow flow of the water through the transducer is difficult to measure and control, and (2) the lucite paddle absorbed water and increased in linear dimensions to such an extent as to strain the semiconductor strain gage beyond its linear range almost to its breaking point. The "zero strain" resistance of the gage had increased nearly 70%.

#### EXPERIMENTAL RESULTS

A thermal transducer using thermistors underwent tests for several months. This design proved to be sensitive to seismic disturbances but the results of many hours of experimentation were unsatisfactory. The thermistors originally used were Victory Engineering Corporation .100 inch

glass probe thermistors with a 1.0 second time constant in still water. It is believed that the long time constant involved made it difficult to detect the normal periodic disturbances associated with S-waves.

The final version of this transducer used .020 inch glass probe thermistors with a 55 millisecond time constant in still water.

We were unable to obtain good results using the thermal transducers because of noise. The noise was probably a result of

- 1) thermal eddy currents in the water due to the heater,
- 2) drift due to changing conditions in the vault, especially temperature change,
- 3) electronic noise in the amplifiers.

A transducer using semiconductor strain gages mounted on a polystyrene paddle was constructed. Results were again unsatisfactory for the following reasons:

- 1) noise due to changing conditions in the vault (drift) and electronic noise in the amplifiers,
- 2) moisture saturation. The paddle absorbed moisture changing the dimensions of the paddle, weakening the bonding material, and penetrating the transducer changing its electrical characteristics and eventually destroying it.

Experimentation is continuing in an attempt to develop a satisfactory transducer to sense the motion of fluid in a circular pipe. The capacitive transducer is being developed further in hopes that it will prove to be the most sensitive transducer to displacement and the least sensitive to noise problems.

PUBLICATION

Rodgers, P. W., "A Note on the Response of the Pendulum Seismometer  
to Plane Wave Rotation," Bull. Seis. Soc. Amer.,  
vol. 59, No. 5, pp. 2101-2102, October 1969.



APPENDIX ATHEORY OF THE ROTATIONAL SEISMOMETER

Consider the simplified view of the rotational seismometer shown in Fig. 1A.

Define the following quantities:

- $a_h$  = cross-sectional area of standpipe
- $a_l$  = cross-sectional area in inertial ring
- $\Omega_z$  = absolute rotation of tube and its mounting frame
- $\varphi$  = absolute rotation of liquid in tube
- $\Psi$  = relative rotation between liquid and tube.

Then

$$\Psi = \Omega_z - \varphi \quad (1A)$$

$\Psi$  and  $z$  are related volumetrically (neglecting compression).

$$z a_h = - \Psi R a_l \quad (2A)$$

The inertial momentum generated in the loop of liquid when the frame undergoes an angular acceleration,  $\ddot{\Omega}_z$ , is opposed by three torques:

- 1) The "spring-like" torque due to the displaced mass,  $z a_h \rho$ , which is acted on by gravity.
- 2) The torque required to accelerate the two liquid masses in the standpipes. This torque is proportional to the mass of the liquid in the standpipes,  $2h a_h \rho$ .

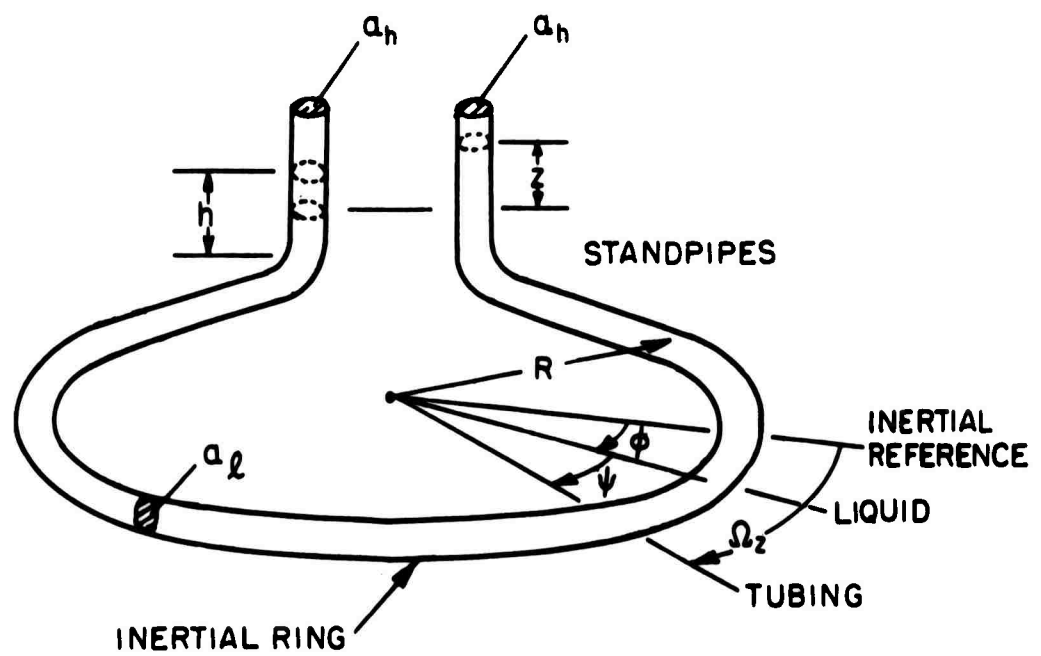


FIG 1A SIMPLIFIED VIEW OF ROTATIONAL SEISMOMETER

- 3) The torque produced by the viscous drag of the liquid which is taken to be proportional to  $\eta(R/a_l)\dot{\psi}$  where  $\eta$  is the viscosity.

From these considerations one can write:

$$\begin{aligned}
 (2\pi R a_l \rho) R^2 \ddot{\phi} &= \underbrace{- 2z a_h \rho \cdot g \cdot R}_{\text{force}} \\
 &\quad \underbrace{- 2h a_h \rho \cdot \ddot{z} \cdot R}_{\text{mass}} \\
 &\quad \underbrace{- \frac{k \eta R}{a_l} \dot{\psi}}_{\text{torque}}
 \end{aligned} \tag{3A}$$

Define Laplace transforms  $\hat{z}(s) = \mathcal{L}[z(t)]$ ,  $\hat{\Omega}_z(s) = \mathcal{L}[\Omega_z(t)]$ , etc.

Substitute (1A) and (2A) into (3A), take Laplace transforms, and solve for the transfer function of the seismometer.

$$\frac{\hat{z}(s)}{\hat{\Omega}_z(s)} = - \frac{2\pi R^2 \left(\frac{a_l}{a_h}\right) s^2}{(2\pi R + 2h)s^2 + BS + g} \tag{4A}$$

$$\frac{\hat{z}(s)}{\hat{\Omega}_z(s)} = \frac{- \frac{2\pi R^2}{2\pi R + 2h} \left(\frac{a_l}{a_h}\right) s^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{5A}$$

where

$$\omega_0 = \sqrt{\frac{g}{2\pi R + 2h}} \tag{6A}$$

and

$$2\zeta\omega_0 = B, \text{ a damping coefficient.}$$

Normally the cross-sectional areas of the standpipes and the ring are equal ( $a_h = a_l$ ) and the height of the standpipes is small compared to the circumference of the ring ( $2h \ll 2\pi R$ ). With these simplifications, the transfer function becomes

$$\frac{\hat{z}(s)}{\hat{\Omega}_z(s)} = - \frac{R s^2}{s^2 + 2\zeta\sqrt{\frac{g}{2\pi R}} s + \frac{g}{2\pi R}} \quad (7A)$$

If the input is considered to be angular acceleration,  $\ddot{\Omega}_z$ , the transfer function is given by:

$$\frac{\hat{z}(s)}{\ddot{\Omega}_z(s)} = \frac{-R}{s^2 + 2\zeta\sqrt{\frac{g}{2\pi R}} s + \frac{g}{2\pi R}} \quad (8A)$$

If  $\Omega_z$  is taken to be sinusoidal in form

$$\Omega_z = \Omega_{z_{\max}} \sin \omega t \quad (9A)$$

Eq. (9A) may be written

$$\frac{z_{\max}}{\Omega_{z_{\max}}}(\omega) = \frac{\omega^2 R}{\left[ \left( \frac{g}{2\pi R} - \omega^2 \right)^2 + 4\zeta^2 \frac{g}{2\pi R} \omega^2 \right]^{1/2}} \quad (10A)$$

This has the same form as the displacement sensitivity of a conventional inertial seismometer. The magnitude of  $1/\omega^2$  times this ratio, which is the acceleration sensitivity, is shown versus frequency in Fig. 2A. Note that the d.c. acceleration sensitivity is simply  $2\pi R^2/g$ . It is

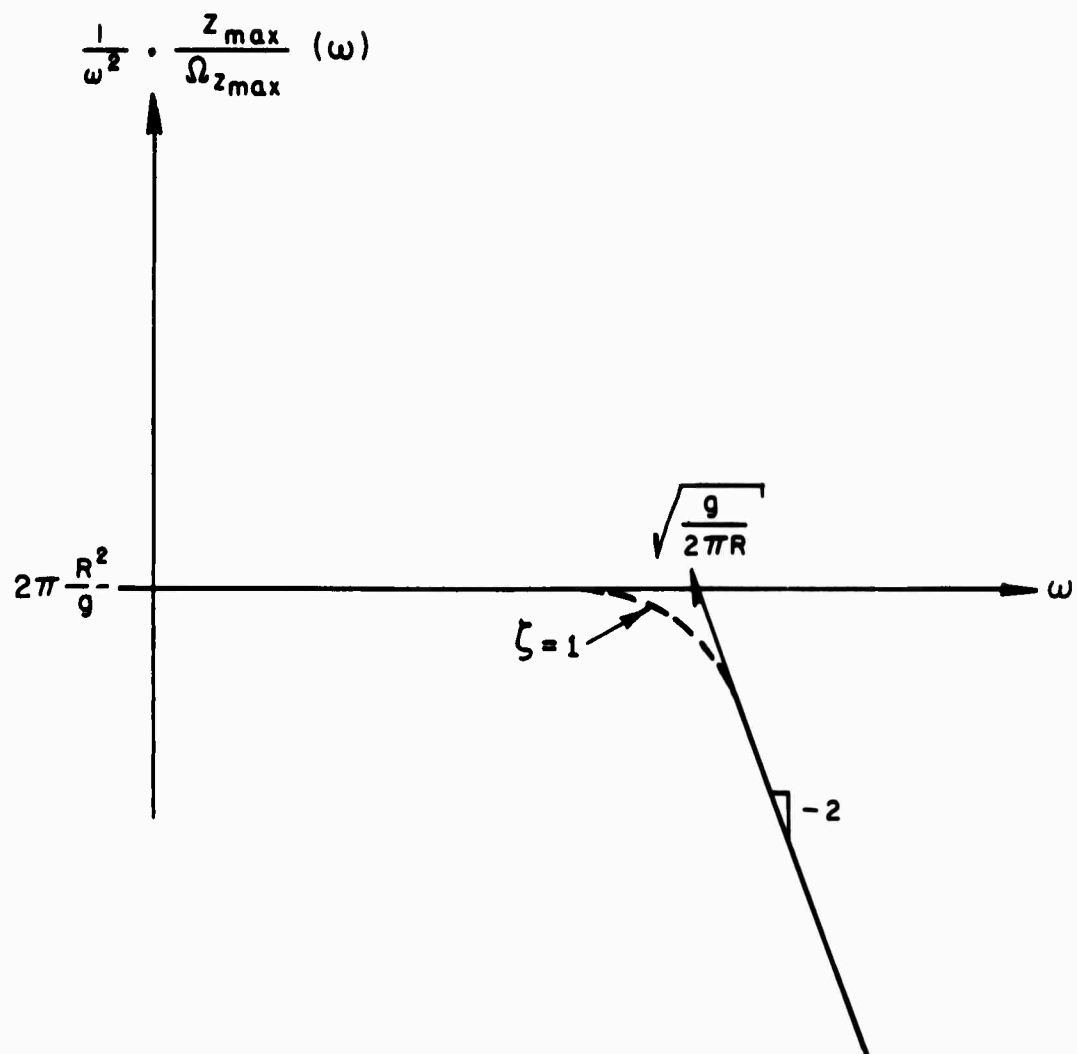
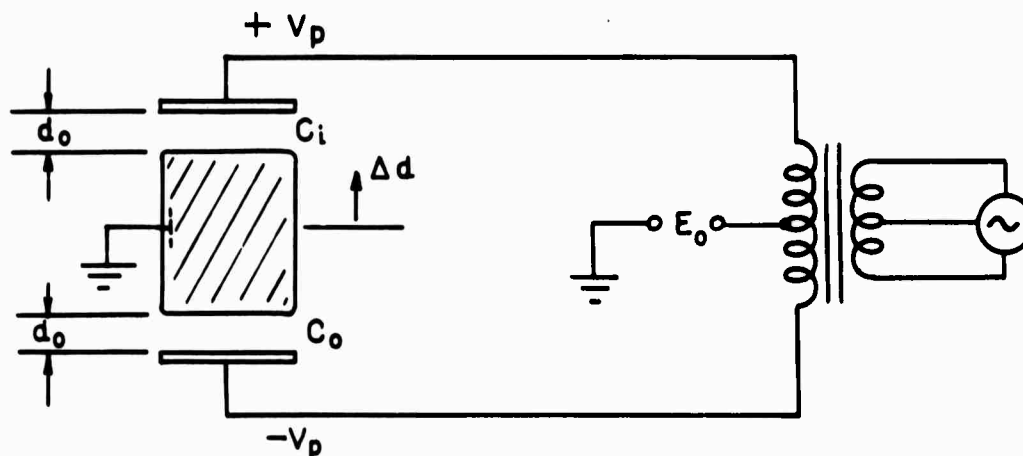


FIG 2A FREQUENCY RESPONSE OF ROTATIONAL SEISMOMETER  
SHOWN ON A LOG-LOG SCALE

also interesting to notice that, except for the damping term, the transfer function of the device is independent of the density of the liquid used to fill the ring and standpipes. Mercury is used in the device described here only because it was available and forms a good meniscus.

APPENDIX B

Bridge output versus mercury movement.



Summing currents at the center tap.

$$E_o \left( \frac{1}{X_1} + \frac{1}{X_o} \right) + \frac{V_p}{X_o} - \frac{V_p}{X_1} = 0 \quad (1B)$$

$$E_o = V_p \frac{X_o - X_1}{X_o + X_1} \quad (2B)$$

Let  $d_o$  be the quiescent spacing of the capacitors. Then if the differential height of the two vertical columns of mercury is  $z$ , each plate is moved an amount  $\Delta d$

$$\Delta d = z/2 \quad (3B)$$

Thus

$$X_1 \propto d_o + \Delta d, \quad X_o \propto d_o - \Delta d \quad (4B)$$

So from (2A)

$$E_o = V_p \frac{\Delta d}{d_o} = 2 \frac{V_p}{d_o} z \quad (5B)$$